

**Counters**

Product code 060681



**An Introduction to Manipulatives**

A manipulative is any object that aids children in visualising mathematical processes. Our range of manipulatives includes Tangrams, Geoboards, Fraction Pieces, Fraction Circles, Fraction Bars, Linking Cubes, Pentominoes and Pattern Blocks. However a manipulative can be as simple as a piece of string or a tin can.

Manipulatives are invaluable in the classroom because, as modern research tells us, children retain information gained from hands-on experiences better than information they gain from memorisation. They learn in a physical way - with their hands as well as their minds. As a physical learning aid, manipulatives encourage this natural learning process by adding a concrete element to ordinarily abstract concepts.

Above all else, children enjoy working with concrete materials - in the hands of young children manipulatives will excite their natural curiosity and motivate them to take responsibility for their own learning. Children will become flexible thinkers with a knowledge of mathematics that can be applied to a wide variety of situations - instead of being taught seemingly unrelated rules, they will learn to be problem solvers.

**Introductory Activities**

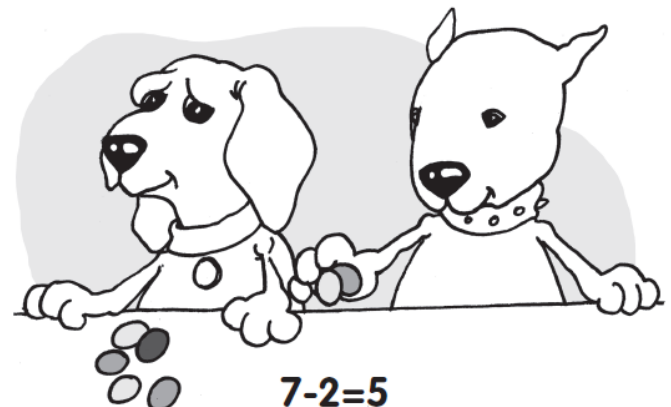
- First, allow the children to engage in free play with the counters. Gradually introduce various fun activities. Familiarity with the counters will help activities go smoother and hopefully reduce the fear of numbers that sometimes develops with traditional maths classes.
- Activities centering around students' ability to estimate can be a good way to introduce counters. Place around 20 counters in a pile. Ask the students if they think they are closer to 10 or 25 counters in the pile. Repeat with different sized piles.
- Show the class a pile of counters. Ask the students if they think there are enough counters for everyone in the class to have one. Ask them to record how many counters they think are in the pile, then count the pile and get the students to check the results against their estimations.

**Conservation**

Counters may be used to explain what is termed "conservation" to younger students. Place a group of counters in a row and ask the student to count them. Rearrange the counters and ask the child how many there are. If the child begins to count the pile again this shows a lack of understanding of the concept of conservation. After a few repetitions of this exercise a child will begin to realise that no matter how we rearrange the counters they will always add up to the same number.

**Addition & Subtraction**

Ask students to use counters to model the story you are telling. "Sam has 6 cars and Joe has 9. How many more does Joe have than Sam?" or "Julie has 8 counters and you have 5. How many more do you need so that you will have the same number?" Note that these are two different ideas. Children need to experience both kinds of comparative subtraction with active lessons and the teacher writing the number sentences before they are asked to deal with them in textbook word problems. Ask students to answer questions using counters. If you have eight counters, you would need how many more to make 10? If you have five counters, how many more would you need to make 10?



## Number Sentences

- Ask each student to model stories with counters. Say something like, "I have seven apples and you take three away. How many will I have left?" After they have used the right number of counters, ask them to write down number sentences to represent stories. Reverse the roles by giving the students a number sentence such as  $4 + 3$  or  $7 - 2$  and asking the students to create an appropriate story and give the solution. Introduce more complex number sentences such as  $3 + 5 + 4$ .
- Give each student ten counters. Ask the children to find three different ways to put them into two sets. Ask them what number sentences they are showing? Are there any other ways to arrange the counters?
- Place six counters of three different colours (a total of 18 counters) in a bag. Have students reach into the bag and remove a handful of counters, sort them by colour and then write the number sentence. Repeat, recording the problem vertically if students first wrote horizontally (or vice versa).

## Multiplication & Division

- When approaching multiplication for the first time, children are required to think of two numbers that represent two different things - the number of groups (or sets) and the number of members in each group. This concept can be made more concrete using counters as groups. Set out some simple multiplications and encourage children to count how many groups and how many counters in each group before counting all of the counters to arrive at the answer. Thus 2 groups of 3 counters is the same as the number 6. And 4 groups of 2 counters is the same as the number 8. Create stories and real life problems to increase interest.
- Take the above exercise one step further by asking the children to set counters out in front of them as you call out various simple multiplications. Perhaps at this stage more specific terminology such as "4 multiplied by 2 counters is?" can be used. Eventually "equals" can be substituted for "is".
- To introduce the concept of division a sharing game can be used. A pair of children can be handed 6 counters and asked to share them equally amongst themselves. Thus 6 counters shared out equally between 2 children means they get 3 each -  $6 \div 2 = 3$ . Repeat the exercise this time asking the children to form groups of 3. This will lead to the realisation that 6 counters between 3 children means 2 counters each -  $6 \div 3 = 2$ . Teachers can then extend the exercise, being sure to use a number of counters that is easily divisible such as 8 and 12 before moving on to remainders.

## Weight



Determine the weight of classroom objects using counters as non-standard units and a balance. Estimate and then check to see how many counters balance the object.

## Sorting Fun

This activity designed to teach children about cooperation and achieving group goals as well as sorting skills.

- 1 Place 4 pieces of coloured paper onto a table.
- 2 Put one container on each piece of paper.
- 3 Give each participating child a random pile of counters. Each student should have between 5 and 20 counters.
- 4 On the count of "1,2,3 Go!" the teacher starts timing and the children try to put each of their counters into the container representing the corresponding colour.
- 5 If someone places the wrong colour in a container they must get it out before they continue.
- 6 If one student finishes first, he or she should help one of the other students place the last of their counters into the containers.
- 7 The timer is stopped when the last counter is placed in its container.
- 8 Be sure to explain to the children that there is no winner in this game - the goal is for the whole group to complete the activity in the shortest time. Cooperation and communication are the key to achieving group goals.

Groups should try to improve their times by investigating different methods for completing the activity. Maybe the children should swap the piles around before the activity starts so that the person nearest to the red container, for example, has the most red counters.

## Area & Perimeter

- The difference between length, perimeter and area can be simply demonstrated using counters. A small sheet of paper can be used as an example. Ask the children to predict how many counters can be laid end-to-end along one of the long sides of the sheet. Then ask them to try it, recording their results. The perimeter can be demonstrated in a similar way.

## Area & Perimeter

- Area can be determined by covering the sheet with counters. In this way simple area calculations for squares and rectangles can be made using “number of counters” as a unit of measurement. It can be pointed out that the number of counters along the length of the sheet multiplied by the number along the width will equal the total number of counters. The students should verify this fact themselves by counting the counters as well as trying the same exercise with different squares and rectangles.

## Probability

- Introduce the concept of probability by asking the class to form pairs and assigning each pair three paper cups (or something similar) and one counter. One student places the counter under a cup while the other is not looking. The other student then tries to guess which cup the counter is under. Ask them to record how many times they are correct and incorrect. Point out that over time they will see a pattern emerge showing that they have a one in three chance of guessing correctly. Reinforce this concept by varying the number of cups and counters.
- Here’s a more complex version of this exercise for teachers and more advanced students.
  - 1 Put 15 counters into a cloth or plastic bag that is not see through. Use 5 counters of 1 colour and 10 of another. Mix them up thoroughly. Don’t let the children see you put the counters into the bag but tell them that you put 5 of 1 colour and 10 of another. Don’t tell them which colour is more common.
  - 2 Pull 3 counters out one at a time, and show them to the class.
  - 3 Tell the children that there are 12 counters left and ask them to predict how many are one colour and how many are the other. Remind them that if they add their predictions together they must equal 12. Ask them to record their predictions.
  - 4 Pull out another 3 counters. Count how many of the 6 are one colour, and how many are the other. Tell them that there are now 9 counters left and, again, ask them to record how many they think are left of each colour.
  - 5 Continue this pattern until all the counters have been taken out of the bag.
  - 6 Point out that the laws of probability tell us that the counters we pull out of the bag give an indication of what colours remain in. As we pull out more counters, our predictions should get better and better.
- Try the exercise again to see if the children have gained an appreciation of probability. Introduce fractions as a convenient notation for probability.
- Repeat the exercise without telling the children the initial ratios.



### THE GAME SHOW PROBLEM

This is a classic mathematical problem for teachers and advanced students. Imagine that the set of a game show has three closed doors. Behind one of these doors is a car; behind the other two are goats. The contestant does not know where the car is, but the host does.

The contestant picks a door and the host opens one of the remaining doors, one he knows doesn’t hide the car. If the contestant has already chosen the correct door, the host is equally likely to open either of the two remaining doors.

After the host has shown a goat behind the door that he opens, the contestant is always given the option to switch doors.

Should she switch or stay with her original choice?

It may seem that the probability that the car is behind each of the remaining doors is  $1/2$ . After all, there are two doors remaining and only one has the car behind it.

However, this is not correct. Originally, we have only 2 distinct sets: the door the contestant has chosen (Door 1) and the doors the contestant has not chosen. Because the probability that the car is behind a given door is  $1/3$  for each door, the probability that it is behind the contestant’s chosen door is  $1/3$ , while the probability that it is behind one of the two remaining doors is  $2/3$ .

When the contestant receives information regarding the doors she has not chosen (the prize cannot be behind Door 3) the probability is still  $2/3$  for the doors she has not chosen. However, the only unknown of these is Door 2 so the probability the car is behind Door 2 is  $2/3$ , therefore the laws of probability state that the contestant should switch.

Test this result by trialing the problem a number of times and recording the results.